

What are the benefits of extended writing in mathematics education?

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In this literature review we aim to understand the place literacy and in particular, extended writing, should hold within mathematics education. More specifically, we seek to understand in what sense is extended writing relevant to the study of mathematics, how can it benefit the development of literacy skills and how can it aid mathematical understanding. Firstly, we will look to understand the role language and expression has within communities of academics and professionals that use mathematics. In doing so we shall argue that importance of training students to be literate in a language style for them to enter such working spheres. Upon doing this, we shall extrapolate a number of benefits of using extended writing in mathematics education for student's literacy skills and their understanding of mathematics. In the process, we shall discuss various theories, such as Vygotsky's (1986) inner speech and the write-to-learn movement that purport that writing is a learning tool. In particular, we shall look at the work of authors such as Pugalee (2004; 2001) and Santos & Semana (2014) who have applied such theories to mathematics education and reported numerous advantages of doing so.

Keywords: Mathematics, extending writing, language, literacy, write-to-learn

Introduction

The relationship between language and learning has received significant interest in research. This has been brought to attention again in recent years in the United Kingdom with concerns regarding the levels of literacy of school and even college leavers (Wilshaw, 2014). Indeed Ofsted have increased their emphasis on literacy across the curriculum (Ofsted, 2013). Furthermore, across the national curriculum for mathematics the "importance of spoken language in pupils' development across the whole curriculum – cognitively, socially and linguistically" (Department of Education, 2014, p. 4; Department of Education, 2014, p. 3; Department of Education, 2013, p. 4) is emphasised. As a part of this drive Ofsted have put a particular focus on writing stating that all subjects should "develop writing skills" (Ofsted, 2013, p. 4) and "use writing as a means of reflecting on and exploring a range of views and perspectives on the world" (Ofsted, 2013, p. 5). Ofsted also suggests that literacy would aid learning in other subjects as well. However, extended writing has traditionally been sparse in

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mathematics classrooms (Morgan, 1998, p. 1) and the writing that has occurred has been heavily procedural consisting of numerical and symbolic calculations (Baroody & Ginsburg, 1990; Nardi & Steward, 2003; Morgan, 1998, p. 1). As such, literacy, and in particular extended writing, can seem disjoint from the practice of mathematics. Moreover, attempts to include extended writing in a mathematics lesson can seem artificial and even pointless by not going beyond practicing basic literacy skills. This prompts the following question which we address in this literature review.

We will show that research indicates that there are many benefits both for literacy and learning mathematics. In fact, we will argue that not only is writing desirable for mathematics education, but it can be viewed as an essential part of it.

To understand the importance of extended writing to learning mathematics we shall initially look at the importance of literacy to the discipline of mathematics. We shall discuss Lave and Wenger's (1991) observation that specific styles and uses of language within a subject play a major role in characterising academic and professional communities that use that subject. It can therefore be argued that not only is it important for students to be literate in such use of language for their future employment prospects, but an education without due regard to such literacy gives an improper reflection of the subject itself!

From establishing the importance of language in mathematics, and using other research, we shall be able to glean many different benefits of the use of extended writing in mathematics education. Firstly, we shall use Lea and Street's (2010) idea of *academic literacy* to suggest that mathematical literacy activities develop not only basic literacy skills but also the student's ability encode and decode information within the context of a particular academic field. Focusing on writing, we shall also discuss the various studies that have been done that suggest that extended writing helps students learn mathematics and solve problems. These are heavily based upon Vygotsky's (1986) theory of inner speech and the *write-to-learn* school of thought. As such, we will give a summary of these theories and their connections to mathematics education. We shall also analyse the work of Pugalee (2001) whose case study suggests extended writing gives an insight into the students' metacognitive processes and hence, can be a unique assessment tool.

It should be noted that though we shall discuss certain kinds of extended writing activities in mathematics, we shall not describe them in great detail. Instead, we shall talk about the relative benefits of certain kinds of writing activities.

Literacy a Natural Part of Mathematical Communities

As in any academic field, communication in mathematics is key for development of the subject. This was observed by work Lave and Wenger (1991) when they introduced the idea of a "community of practice" which they defined as "a system of relationships between people, activities, and the world; developing with time, and in relation to other tangential and overlapping communities of practice" (1991, p. 98). They state that the use of language is a key characteristic of any such community since different communities have different literary styles and its own jargon. As a result of this, if one is to become an expert member of the community, they must gain fluency in its language style (Lave & Wenger, 1991). Lea and Street (2010) and Seligmann (2011) echo this thought by suggesting each academic field has its own form of literacy. Therefore, if we are to view classrooms as preparing or even being a part of a wider academic community, the language of that community needs to be an intrinsic part of the classroom.

Moreover, with Lave and Wenger's (1991) conceptualisation of communities of practice, it becomes clear that developing in an academic language style gives the learner greater sense of belonging in that academic community. Lea and Street (2010) argue that this identification as a member of the community helps pupils feel empowered and gives them ownership over their learning. In particular, Johnston-Wilder and Lee (2008) stress that by wrestling mathematical

language within a context where pupils view themselves as mathematicians develops their academic resilience. Evidence for this claim is given in a case study by Goos (2004). Monroe (1996) argues that not only is the mathematical language is key for the mathematical communities to communicate ideas but can be a tool by which they create new ones. She claims that foundational mathematical ideas can arise naturally from contexts that are formed in discourse and language. As an example, she points out that the idea of geometry naturally arises when pupils asked to place a book in the middle of a table. With this in mind, the mathematical language becomes elevated from just a means of communication for the student to an actual learning tool.

Benefits for literacy

There is evidence to suggest that including literacy activities in other subjects on which students are given effective feedback could improve their literacy skills. The increased opportunity to practice literacy should, under the constructivist model of learning proposed by Piaget (1965), lead to development in literacy skills. Though authors such Willingham (2009) and Ericsson (2004; 1993) do support this idea, they point out that experience alone does not lead to expert performance. Ericsson's influential work with his colleagues on expertise acquisition showed that expert performance is attained when, what they call, *deliberate practice* is employed. Ericsson characterises this as practice where "individuals [are] 1) given a task with a well-defined goal, 2) motivated to improve, 3) provided with feedback, and 4) provided with ample opportunities for repetition and gradual refinements of their performance" (Ericsson, 2004, p. 991). Indeed, from their observations of schools with good literacy outcomes, Ofsted recommend "embedding good practice in schemes of work and development planning" and "systematic and effective monitoring and evaluation" (Ofsted, 2013, p. 40). Moreover, having a cross-curricular literacy strategy meets Ofsted's wider definition of literacy that goes beyond "mechanics of reading, writing, speaking and listening" (Ofsted, 2013, p. 5). For Ofsted it also entails:

"that connections be made between each strand and across subjects, which calls for thought and understanding, for recall, selection and analysis of ideas and information, and for coherent, considered and convincing communication in speech and in writing" (Ofsted, 2013, p. 5)

This can be thought of as what Boomer (1985) describes as "active literacy" which, as well as encoding and decoding information into the syntax of the language, requires a person assimilating the information into their existing body of knowledge. Such a definition is theoretically supported by Halliday's claim that language is "the prototypical form of human semiotic" (Halliday, 1993, p. 93) which, in other words means that language is the fundamental process by which we make meaning. Thus, to become literate in this sense the pupil is required to be able to extract information from the various different ways that language may be used. Since different academic disciplines use language in different ways it therefore becomes necessary for pupils to experience these different literacy forms in different subjects to achieve active literacy.

Unique Aspects of the Mathematical Language the Support Literacy Development

There are various components of mathematical language, many of which can possibly help general literacy skills. One of the most unique characteristics of mathematical language is its symbolic content. This aspect has been studied by Ervick (Mathematics as a foreign language, 1992) and Kane (1967) with Kane even defining "mathematical English" as "a hybrid

language...composed of ordinary english commingled with various brands of highly stylized formal symbol systems” (1967, p. 296). These formal symbol systems indeed have their own grammar i.e. rules for syntax and of course their semantics are derived from the context (Kane, 1967).

Morgan (1998) criticised this approach for not appreciating that “the non-symbolic ‘ordinary’ component also has specifically mathematical aspects”. Upon similar observations, Halliday (1975) introduced what he calls the *mathematical register*, a notion elaborated on by Pimm (1987). Halliday describes a *register* as “set of meanings that is appropriate to the particular function of the language, together with the words and structures which express these meanings” (Halliday, 1975, p. 65). The meanings in the mathematical register are extremely strict and can differ considerably from the meaning in natural language (Pimm, 1987, p. 78). Pimm explains how these two factors result in significant scope for misunderstanding for the pupil that has not grasped the register fully (1987, pp. 88-93). So in this sense, by engaging with the mathematical register pupils are learning to appreciate precision of meaning in certain formal contexts.

However, Morgan highlights that “it is not clear that the idea of a single mathematical register is sufficient to cope with the variation of functions and meanings” (Morgan, 1998, p. 10). She draws upon work of Richards’ (1991) who identifies different ‘domains of discourse’ within mathematics such as that written in journals and spoken by mathematicians. Richards’ claims that between two such domains not only may the subject matter be different but the very nature argumentation as well. Sipka (1990) identifies further different categories that tend to occur in written school mathematics, namely-

- Formal activities which include proof, paper writing, lecture notes and even writing letters to authors.
- Informal activities which include mathematical autobiographies, journals, free writing and reading logs.

Hence, as pupils learn mathematical literacy they are required to navigate between what Lea and Street (2010) call *genres* meaning different styles of mathematical text. As a result, they exercise the very skills extracting and translating information encoded in different text that are required by the above definition of literacy.

Benefits for Learning Mathematics

As indicated in the introduction, exercising literacy within mathematics can help learning of mathematics also. Namely, we stated that mathematical literacy enables pupils to engage with literature, lectures and discourse (both written and oral) within a discipline. We also discussed how deficiencies in aspects of the mathematical literacy, in particular the mathematical register, can obscure mathematical understanding. Many have suggested various other benefits of literary exercises in mathematics which will be discussed in the following section.

Impact of Language on Thought

These benefits rest upon language being a conduit of thought (Halliday, 1993). However, there is a large school of thought that believes in a converse relationship, i.e. that language impacts thought (see Gleitman & Papafragou, 2012). It has been suggested by linguistic anthropologists such as Whorf (1956) and Sapir (1929) that language not only conveys thought but it shapes it too. This is in contrast with Piaget (1965) who believed that thought preceded language. However, this in turn was rebutted by Vygotsky (1986) who, in his landmark work *Thought and Language* presents the idea of inner speech. Vygotsky says the following:

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"Inner speech is not the interior aspect of external speech - it is a function in itself. It still remains speech, i.e. thought connected with words. But while in external speech thought is embodied in words, in inner speech words dies as they bring forth thought. Inner speech is to a large extent thinking in pure meanings" (1986, p. 149)

Various theorists of education derived pedagogies and theories of thought upon Vygotsky's idea of inner speech. Notably, Alexander (2006) proposes a pedagogy based on dialogue in the classroom and Sfrad who defines thinking to be "an individualized version of (interpersonal) communication" (2010, p. 81).

The Role of Writing

Vygotsky commented on the importance of writing as a means of expressing inner speech because he viewed it as a deliberate act of making meaning and because he believed it was the "maximally detailed" form of speech (1986, p. 100). On this basis Britton et al. (1975) argued that writing enables people to access inner speech. Britton also claims that writing helps organise experience (Britton, 1970). Emig (1971) built on this idea and argued that writing is a unique learning tool because it connects cognitive and physical means of interrogating, connecting and reviewing ideas. It is thought that the work of Emig and Britton et al. gave rise to the *Write-to-Learn* movement which emphasised writing as a means for learning (Bazerman, et al., 2005).

This movement has come into criticism in the past for lacking empirical evidence to support its claim (Morgan, 1998, p. 25; Shield & Galbraith, 1998). However, over past 15 years there have been a number of case studies on extended writing in mathematics education which report various benefits (Baxter, et al., 2005; Pimm, 1987; Pugalee, 2004; Pugalee, 2004; Pugalee, 2001; Santos & Semana, 2014). There is now a considerable body of evidence to support the idea of writing as a learning tool (for a good overview of the research see Klein & Boscolo, 2016).

Most of the case studies have sought to use writing to aid higher order thinking and access what Skemp (Skemp, 2002) famously called *relational knowledge*. Skemp defined this the knowledge of relationships between ideas being presented i.e. the *why* behind an idea. The traditional writing in mathematics classrooms mentioned in the introduction has only accessed what Skemp (Skemp, 2002) calls *instrumental knowledge* i.e. *how* a process or technique works. Skemp explains that though instrumental knowledge has the benefit of being easier to apply and gives quick rewards, he goes on to explain that relational knowledge is more adaptable and endows the pupils with more transient problem solving skills (Skemp, 2002, pp. 8-10).

Writing Exercises that Access Higher Order Thinking

There are also various forms of informal writing that have been found to access higher order and relational thinking. One of the most popular is that of mathematics journals which are the student's own log of mathematical thought on the material they are being taught. These offer pupils the opportunity to reflect on their learning. Many benefits of such journals have been reported. For instance Waywood (1994), Powell & Ramnauth (1992) and Powell & Lopez (1989) report significant improvements in questions posed (Waywood, 1994). Moreover, Powell and Lopez (1989) claim that over time pupils' writing became more reflective. Powell and Ramnauth (1992) suggest that there is increased confidence in expression of ideas. Furthermore, Borasi and Rose's case study (Borasi & Rose, 1989, p. 352) suggest that articulation of this reflection in journals also leads to:

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- “A therapeutic effect on the emotional components of learning mathematics can result as students express and reflect on their feelings about the course, mathematics and schooling”
- “Increased knowledge of mathematical content”.
- “Improvement in learning and problem solving skills”
- “Steps towards achieving a more appropriate view of mathematics”. (1989, p. 352)

Another informal writing exercise that has been studied by Pugalee (2004) is that of writing “global plans” to a mathematical problem before attempting to solve it. In this study Pugalee found that pupils that gave an oral or written description of global plans were more successful in solving the problem. Furthermore, he reports that writing had a more significant effect than orally verbalising.

There are relatively fewer forms of formal writing which have been investigated in the literature. Nevertheless, Grossman et al. (1993) did investigate a certain kind of formal questioning in which students had to compare, contrast and describe different mathematical processes. They concluded from their findings that “a student’s ability to explain concepts is related to the student’s ability to comprehend and apply a concept” (Grossman, et al., 1993, p. 4). Also, Santos and Semana (2014) conducted a study of formal expository writing also document that greater “the elaborateness of students’ expository writing” (2014, p. 84).

Writing as an Assessment Tool

Aside from the benefits for learning, there are many claims that have been made about extending written exercises in mathematics as an assessment tool (Burns, 2014; Pugalee, 2001; Morgan, 1998, pp. 115-121; Baxter, et al., 2005; Borasi & Rose, 1989; Santos & Semana, 2014). For instance, Pugalee (2001) evidences that writing descriptions of global plans to mathematics problems exhibit their “metacognitive framework” and hence give a unique insight into a pupil’s understanding of relational knowledge. Various authors have also reported the benefits of journal writing in mathematics. Borasi and Rose claim journals give the educators:

- “More appropriate evaluation and remediation of individual students” (1989, p. 353)
- “Immediate changes and improvements in the course” (1989, p. 353)
- “Long-term improvements in teaching approach and methodologies” (1989, p. 353)
- “More individualise teaching can be achieved” (1989, p. 353)
- “A more caring and non-adversarial classroom atmosphere” (1989, p. 353)

Baxter et al. (2005) give evidence that of attributes of journals significantly aid teachers in supporting mathematically low achieving pupils.

Following findings from students’ writing with effective feedback can result in improved understanding and literacy skills. Indeed, Santos and Semana (2014) found that using formative assessment techniques with students’ expository writing resulted in “more relational justifications, instead of vague statements, rules or procedural descriptions when we compare first and second drafts of each” (Santos & Semana, 2014, p. 65). This harkens back to Ericsson’s (2004) model of ‘deliberate practice’ mentioned above wherein prompt and precise feedback is essential for a learner to make rapid progress.

It should be noted that the findings of Santos and Semana contradict those of Shield and Galbraith (1998). However, as Santos and Semana explain, this maybe a result of differing aims in the two studies; Shield and Galbraith sought to present “scheme for coding the parts of written mathematical presentations” whereas Santos and Semana’s main pedagogical aim was to improve expository writing.

Conclusion

The work of Lave and Wenger (1991) highlights that the way in which language is used of language is at the very heart of how academic and professional communities function. As such, it becomes not only desirable to teach subject based language, but it is important to do so.

Evidence shows a myriad of possible benefits for the pupil from engaging with academic literacy and, in particular, writing within mathematics. These include -

1. Improved literacy, both mechanical and active. In particular, some of the unique aspects of the mathematical language, such as the syntax in its symbol system or the preciseness of terms, may offer students a different way to study aspects of linguistics.
2. Confidence and resilience in the subject through deeper identification as member of that academic community.
3. Being able to access a wider scope of discourse in the subject.
4. Literacy, and in particular writing, aiding metacognitive processes.
5. Written tasks can offer a unique assessment tool for the teacher.

In spite of the large body of research advocating the use of writing in mathematics, surprisingly little extended writing is used in British mathematics classrooms (Baroody & Ginsburg, 1990; Nardi & Steward, 2003). Studies have indicated this is much to do with teachers' belief of what mathematics is and, hence, how it is best learned (Kenney, et al., 2014; Kuzle, 2013). It is also recognised that the nature of high-stakes exams has a large part in these beliefs (e.g. see Willis, 2007; Polesel, et al., 2014). Therefore, more work needs to be done to investigate how exams can be altered so as to encourage writing. It could be argued that this process has already begun since the new mathematics GCSE and A-Level have placed greater emphasis on problem solving. Therefore, a very real and current research question would be, "what writing strategies improve success rates of solutions to GCSE problem solving questions?" Studies from Kenny et al. (2014) and Kulze (2013) suggest that if a trainee teacher is asked to utilise extended writing then many tend see the benefits of it causing those that were against it to reconsider their beliefs. This suggests that it would be worth considering writing strategies are taught to trainee teachers.

On another note, involving extended writing in mathematics poses an interesting question for student engagement which, considering the low uptake in the subject despite the high demand (Boaler, 2009), is always a pressing issue. In view of the difference of extended writing to traditional activities in mathematics classrooms, would extended writing engage a new group of students who enjoy writing? On the other hand, would it disengage students who do enjoy mathematics lessons? It is thought by some that those students that enjoy mathematics are often those that do not enjoy writing. Thus, we can ask whether there are extended writing tasks that engage a new group of students yet do not disengage students who traditionally enjoy mathematics and still attains the benefits above.

The current body of research seems to strongly suggest that writing as a learning tool holds numerous benefits (Klein & Boscolo, 2016). Thus, the question shifts from "whether writing is beneficial for mathematics education" to "how can we best extrapolate benefits of extended writing in mathematics". In view of this, the key issue now revolves around implementation of writing strategies. The increased focus on problem solving on all levels of the curriculum may provide fertile ground to do investigate this problem. Whatever the best implementation methods may be, extended writing could prove to be a powerful tool to increase problem solving skills and increase engagement with the subject.

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